

Calcola il valore dell'espressione applicando le proprietà delle potenze.

Fila A

$$\begin{aligned}
 & \bullet \left[\left(1 - \frac{1}{4} - \frac{5}{12} \right)^3 : \left(\frac{1}{3} - \frac{1}{2} \right)^2 - \left(\frac{3}{2} \right)^{-2} \right] : \left\{ \left[(-4)^{-3} : \left(\frac{1}{8} - \frac{3}{4} \right) - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \left[\left(\frac{12-3-5}{12} \right)^3 : \left(\frac{2-3}{6} \right)^2 - \left(\frac{2}{3} \right)^2 \right] : \left\{ \left[\left(-\frac{1}{4} \right)^3 : \left(\frac{1-6}{8} \right) - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \left[\left(\frac{4}{12} \right)^3 : \left(-\frac{1}{6} \right)^2 - \left(\frac{2}{3} \right)^2 \right] : \left\{ \left[\left(-\frac{1}{4} \right)^3 : \left(-\frac{5}{8} \right) - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \left[\left(\frac{1}{3} \right)^3 : \left(\frac{1}{6} \right)^2 - \frac{4}{9} \right] : \left\{ \left[-\frac{1}{64} \cdot \left(-\frac{8}{5} \right) - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \left[\frac{1}{27} \cdot 36 - \frac{4}{9} \right] : \left\{ \left[-\frac{1}{8} \cdot \left(-\frac{1}{5} \right) - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \left[\frac{4}{3} - \frac{4}{9} \right] : \left\{ \left[\frac{1}{40} - \frac{2}{5} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \left[\frac{12-4}{9} \right] : \left\{ \left[\frac{1-16}{40} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \left[\frac{8}{9} \right] : \left\{ \left[-\frac{15}{40} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \left[\frac{8}{9} \right] : \left\{ \left[-\frac{3}{8} \right]^2 \cdot \left(-\frac{4}{3} \right)^3 \right\} = \\
 & = \frac{8}{9} : \left\{ \left[-\frac{3}{2^3} \right]^2 \cdot \left(-\frac{2^2}{3} \right)^3 \right\} = \frac{8}{9} : \left\{ \frac{3^2}{2^6} \cdot \left(-\frac{2^6}{3^3} \right) \right\} = \frac{8}{9} : \left\{ \frac{1}{1} \cdot \left(-\frac{1}{3} \right) \right\} = \\
 & = \frac{8}{9} : \left\{ -\frac{1}{3} \right\} = -\frac{8}{9} \cdot 3 = -\frac{8}{3}
 \end{aligned}$$

Fila B

$$\begin{aligned}
 & \bullet \left\{ \left[(-3)^2 : \left(\frac{13}{15} + \frac{1}{3} \right) + 2^{-1} \right]^2 : (-2)^9 \right\} : \left\{ 1 - (-4)^{-3} : \left[\frac{5}{8} - \left(\frac{3}{7} \right) : \left(-\frac{3}{7} \right) - \frac{3}{2} \right] \right\} = \\
 & = \left\{ \left[9 : \left(\frac{13+5}{15} \right) + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 - (-4)^{-3} : \left[\frac{5}{8} - (-1) - \frac{3}{2} \right] \right\} = \\
 & = \left\{ \left[9 : \left(\frac{18}{15} \right) + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 - (-4)^{-3} : \left[\frac{5}{8} + 1 - \frac{3}{2} \right] \right\} = \\
 & = \left\{ \left[9 : \left(\frac{6}{5} \right) + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 - (-4)^{-3} : \left[\frac{5+8-12}{8} \right] \right\} = \left\{ \left[9 \cdot \frac{5}{6} + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 - (-4)^{-3} : \left[\frac{1}{8} \right] \right\} =
 \end{aligned}$$

$$\begin{aligned}
&= \left\{ \left[3 \cdot \frac{5}{2} + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 - \left(-\frac{1}{4^3} \right) : \left[\frac{1}{2^3} \right] \right\} = \left\{ \left[\frac{15}{2} + \frac{1}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 + \frac{1}{(2^2)^3} : \left[\frac{1}{2^3} \right] \right\} = \\
&= \left\{ \left[\frac{16}{2} \right]^2 : (-2)^9 \right\} : \left\{ 1 + \frac{1}{2^6} : \left[\frac{1}{2^3} \right] \right\} = \left\{ [8]^2 : (-2)^9 \right\} : \left\{ 1 + \frac{1}{2^3} \right\} = \left\{ [2^3]^2 : (-2)^9 \right\} : \left\{ 1 + \frac{1}{2^3} \right\} = \\
&= \left\{ 2^6 : (-2)^9 \right\} : \left\{ 1 + \frac{1}{8} \right\} = \left\{ -2^{-3} \right\} : \left\{ 1 + \frac{1}{8} \right\} = \left\{ -\frac{1}{8} \right\} : \left\{ \frac{8+1}{8} \right\} = \left\{ -\frac{1}{8} \right\} : \left\{ \frac{9}{8} \right\} = \\
&= \left\{ -\frac{1}{8} \right\} \cdot \left\{ \frac{8}{9} \right\} = -\frac{1}{9}
\end{aligned}$$

Semplifica l'espressione.

Fila A

$$\begin{aligned}
&\bullet \left[\left(3x^2y - \frac{5}{2}x^2y \right)^3 : \left(-\frac{1}{4}x^4y^2 \right) - \left(+\frac{1}{3}y \right) (-x^2) \right] (-xy^2) + \left(\frac{1}{2}xy \right) \left[\frac{5}{6}xy + \frac{1}{6}xy \right]^2 = \\
&= \left[\left(\frac{1}{2}x^2y \right)^3 : \left(-\frac{1}{4}x^4y^2 \right) + \frac{1}{3}x^2y \right] (-xy^2) + \left(\frac{1}{2}xy \right) [xy]^2 = \\
&= \left[\left(\frac{1}{2^3}x^6y^3 \right) : \left(-\frac{1}{2^2}x^4y^2 \right) + \frac{1}{3}x^2y \right] (-xy^2) + \left(\frac{1}{2}xy \right) x^2y^2 = \left[-\frac{1}{2}x^2y + \frac{1}{3}x^2y \right] (-xy^2) + \frac{1}{2}x^3y^3 = \\
&= \left[\frac{-3+2}{6}x^2y \right] (-xy^2) + \frac{1}{2}x^3y^3 = \left[-\frac{1}{6}x^2y \right] (-xy^2) + \frac{1}{2}x^3y^3 = \frac{1}{6}x^3y^3 + \frac{1}{2}x^3y^3 = \\
&= \frac{1+3}{6}x^3y^3 = \frac{4}{6}x^3y^3 = \frac{2}{3}x^3y^3
\end{aligned}$$

Fila B

$$\begin{aligned}
&\bullet \left\{ \left(+\frac{7}{2}xy^3 \right) \left[+\frac{1}{4}(-xy)^2 \right] + \left(-\frac{3}{2}xy \right) (-y)^2 \right\} : \left(-\frac{5}{4}xy^4 \right) + \left(\frac{7}{4}xy - \frac{11}{4}xy \right)^2 : (-y) = \\
&= \left\{ \left(+\frac{7}{2}xy^3 \right) \left[+\frac{1}{4}x^2y^2 \right] - \frac{27}{8}x^3y^3 \cdot y^2 \right\} : \left(-\frac{5}{4}xy^4 \right) + \left(\frac{7-11}{4}xy \right)^2 : (-y) = \\
&= \left\{ \frac{7}{8}x^3y^5 - \frac{27}{8}x^3y^5 \right\} : \left(-\frac{5}{4}xy^4 \right) + \left(-\frac{4}{4}xy \right)^2 : (-y) = \\
&= \left\{ -\frac{20}{8}x^3y^5 \right\} : \left(-\frac{5}{4}xy^4 \right) + (-xy)^2 : (-y) = \left\{ -\frac{5}{2}x^3y^5 \right\} : \left(-\frac{5}{4}xy^4 \right) + x^2y^2 : (-y) = \\
&= \frac{5}{2} \cdot \frac{4}{5}x^2y - x^2y = 2x^2y - x^2y = x^2y
\end{aligned}$$

Utilizza i prodotti notevoli per semplificare l'espressione.

Fila A

$$\begin{aligned} & \bullet \quad (a+b^2)^3 - b^4 \left[3 \left(a - \frac{1}{2} \right) + (b+1)(b-1) \right] + \left(-\frac{3}{2} \right) (a^2 + b^2)^2 = \\ & = a^3 + 3a^2b^2 + 3ab^4 + b^6 - b^4 \left[3a - \frac{3}{2} + b^2 - 1 \right] + \left(-\frac{3}{2} \right) (a^4 + 2a^2b^2 + b^4) = \\ & = a^3 + 3a^2b^2 + 3ab^4 + b^6 - 3ab^4 + \frac{3}{2}b^4 - b^6 + b^4 - \frac{3}{2}a^4 - 3a^2b^2 - \frac{3}{2}b^4 = \\ & = -\frac{3}{2}a^4 + a^3 + b^4 \end{aligned}$$

Fila B

$$\begin{aligned} & \bullet \quad 12a(a-b) \left[\frac{1}{2}(a+b) \right] + b \left[(b+2a)^2 + 2a(4a-2b) \right] + (2a-b)^3 = \\ & = 6a(a-b)(a+b) + b \left[b^2 + 4ab + 4a^2 + 8a^2 - 4ab \right] + 8a^3 - 12a^2b + 6ab^2 - b^3 = \\ & = 6a(a^2 - b^2) + b \left[b^2 + 12a^2 \right] + 8a^3 - 12a^2b + 6ab^2 - b^3 = 6a^3 - 6ab^2 + b^3 + 12a^2b + 8a^3 - 12a^2b + 6ab^2 - b^3 = \\ & = 14a^3 \end{aligned}$$

Scomponi in fattori il seguente polinomio, raccogliendo a fattor comune un monomio.

Fila A

$$\bullet \quad 4x^2y^2 - 6x^3y + 8x^2y^3 = 2x^2y(2y - 3x + 4y^2)$$

Fila B

$$\bullet \quad 9a^3b^4 - 12a^2b^3 + 3a^2b^2 = 3a^2b^2(3ab^2 - 4b + 1)$$

Scomponi in fattori la seguente espressione algebrica, raccogliendo a fattor comune un polinomio.

Fila A

$$\bullet \quad (a-3)(2a-4) - (a-2)(a-3) = (a-3)(2a-4-a+2) = (a-3)(a-2)$$

Fila B

$$\bullet \quad (3x+2)(2x+2) - (3x+2)(x+1) = (3x+2)(2x+2-x-1) = (3x+2)(x+1)$$

Scomponi in fattori con il metodo del raccoglimento parziale.

Fila A

$$\begin{aligned} & \bullet \quad ax^2 - ab^2 + b^2x - x^3 = \\ & \quad = a(x^2 - b^2) - x(x^2 - b^2) = (x^2 - b^2)(a - x) = (x-b)(x+b)(a-x) \end{aligned}$$

$$\bullet \quad (y^2 - y)^2 - 7y^2 + 7y = y^2(y-1)^2 - 7y(y-1) = y(y-1)[y(y-1) - 7] = y(y-1)(y^2 - y - 7)$$

Fila B

- $4 - a^2x^3 + 2ax - 2ax^2 =$
 $= 4 - a^2x^3 + 2ax - 2ax^2 = 2(2 + ax) - ax^2(ax + 2) = (2 + ax)(2 - ax^2)$
- $(2a - b)^2 - 4a^2 + 2ab = (2a - b)^2 - 2a(2a - b) = (2a - b)(2a - b - 2a) = -b(2a - b) = b(b - 2a)$

Scomponi in fattori, dopo aver osservato che ciascun polinomio è la differenza di due quadrati.

Fila A

- $a^2 - 64b^2 = (a - 8b)(a + 8b)$
- $16x^4 - y^4 = (4x^2 - y^2)(4x^2 + y^2) = (2x - y)(2x + y)(4x^2 + y^2)$

Fila B

- $36x^2 - y^2 = (6x - y)(6x + y)$
- $a^4 - 81b^4 = (a^2 - 9b^2)(a^2 + 9b^2) = (a - 3b)(a + 3b)(a^2 + 9b^2)$

Scomponi in fattori, dopo aver osservato che ciascun polinomio è il quadrato di un binomio.

Fila A

- $9x^2 - 6xy + y^2 = (3x - y)^2$
- $\frac{1}{2}a^2 + 4a + 8 = \frac{1}{2}(a^2 + 8a + 16) = \frac{1}{2}(a + 4)^2$

Fila B

- $a^2 - 8ab + 16b^2 = (a - 4b)^2$
- $\frac{1}{3}x^2 + 3 - 2x = \frac{1}{3}(x^2 + 9 - 6x) = \frac{1}{3}(x - 3)^2$

Scomponi in fattori, riconoscendo il cubo di un binomio.

Fila A

- $-27xy^2 - x^3 + 9x^2y + 27y^3 = (3y - x)^3$

Fila B

- $-a^3 + 6a^2b - 12ab^2 + 8b^3 = (2b - a)^3$

Scomponi in fattori, riconoscendo la somma o la differenza di due cubi.

Fila A

- $x^3y^6 - 125 = (xy^2 - 5)(x^2y^4 + 5xy^2 + 25)$
- $\frac{2}{3}a^3 + \frac{16}{81}b^9 = \frac{2}{3}\left(a^3 + \frac{8}{27}b^9\right) = \frac{2}{3}\left(a + \frac{2}{3}b^3\right)\left(a^2 - \frac{2}{3}ab^3 + \frac{4}{9}b^6\right)$

Fila B

- $8 - a^9 b^3 = (2 - a^3 b)(4 + 2a^3 b + a^6 b^2)$
- $\frac{81}{16}x^3 - \frac{3}{2}y^6 = \frac{3}{2}\left(\frac{27}{8}x^3 - y^6\right) = \frac{3}{2}\left(\frac{3}{2}x - y^2\right)\left(\frac{9}{4}x^2 + \frac{3}{2}xy^2 + y^4\right)$

Scomponi in fattori i seguenti trinomi particolari.

Fila A

- $x^2 - 2x - 48 = (x - 8)(x + 6)$
- $a^2 - 5a - 36 = (a - 9)(a + 4)$

Fila B

- $x^2 + 3x - 28 = (x + 7)(x - 4)$
- $x^2 - x - 30 = (x - 6)(x + 5)$

Esegui l'espressione e semplifica il risultato, se è possibile.

Fila A

- $\frac{x+1}{x-2} - \frac{5-3x}{x+3} - \frac{3x^2+7}{x^2+x-6} =$ C.E.: $x-2 \neq 0 \Rightarrow x \neq 2$; $x+3 \neq 0 \Rightarrow x \neq -3$

$$= \frac{x+1}{x-2} - \frac{5-3x}{x+3} - \frac{3x^2+7}{(x+3)(x-2)} = \frac{(x+1)(x+3) - (5-3x)(x-2) - (3x^2+7)}{(x+3)(x-2)} =$$

$$= \frac{x^2+3x+x+3 - (5x-10-3x^2+6x) - 3x^2-7}{(x+3)(x-2)} = \frac{x^2+3x+x+3-5x+10+3x^2-6x-3x^2-7}{(x+3)(x-2)} =$$

$$= \frac{x^2-7x+6}{(x+3)(x-2)} = \frac{x^2-7x+6}{x^2+x-6}$$

Fila B

- $\frac{x^2+5}{x^2-x-2} + \frac{x+1}{2-x} - \frac{x-2}{x+1} =$ C.E.: $x-2 \neq 0 \Rightarrow x \neq 2$; $x+1 \neq 0 \Rightarrow x \neq -1$.

$$= \frac{x^2+5}{(x-2)(x+1)} - \frac{x+1}{x-2} - \frac{x-2}{x+1} = \frac{x^2+5 - (x+1)(x+1) - (x-2)(x-2)}{(x-2)(x+1)} = \frac{x^2+5 - (x+1)^2 - (x-2)^2}{(x-2)(x+1)} =$$

$$= \frac{x^2+5 - (x^2+2x+1) - (x^2-4x+4)}{(x-2)(x+1)} = \frac{x^2+5-x^2-2x-1-x^2+4x-4}{(x-2)(x+1)} = \frac{2x-x^2}{(x-2)(x+1)} =$$

$$= \frac{-(x^2 - 2x)}{(x-2)(x+1)} = \frac{-x(x-2)}{(x-2)(x+1)} = -\frac{x}{x+1}$$

Semplifica l'espressione (le condizioni di esistenza sono soddisfatte).

Fila A

$$\begin{aligned} & \bullet \quad \frac{b^2 - 4}{b^2 - 2b + 1} \cdot \left(\frac{b-1}{b^2 - 3b + 2} \right) : \left(\frac{b-5}{b^2 - 6b + 5} \right) \cdot \frac{1}{b^2 + 4b + 4} = \\ &= \frac{(b+2)(b-2)}{(b-1)^2} \cdot \frac{b-1}{(b-2)(b-1)} : \frac{b-5}{(b-5)(b-1)} \cdot \frac{1}{(b+2)^2} = \\ &= \frac{(b+2) \cancel{(b-2)}}{(b-1)^2} \cdot \frac{\cancel{b-1}}{\cancel{(b-2)} \cancel{(b-1)}} : \frac{b-5}{(b-5)(b-1)} \cdot \frac{1}{(b+2)^2} = \frac{b+2}{(b-1)^2} \cdot \frac{(b-5)(b-1)}{b-5} \cdot \frac{1}{(b+2)^2} = \\ &= \frac{b+2}{(b-1)^2} \cdot \frac{\cancel{(b-5)} \cancel{(b-1)}}{\cancel{b-5}} \cdot \frac{1}{(b+2)^2} = \frac{\cancel{b+2}}{b-1} \cdot \frac{1}{(b+2)^2} = \frac{1}{(b-1)(b+2)} \end{aligned}$$

Fila B

$$\begin{aligned} & \bullet \quad a \left(1 + \frac{a}{2b} \right) \left(1 - \frac{a}{2b} \right) : \frac{a^2}{2b} : \left(\frac{a+2b}{a} \cdot \frac{2b-a}{2b} \right) = \\ &= a \left(1 - \frac{a^2}{4b^2} \right) : \frac{a^2}{2b} : \left(-\frac{a+2b}{a} \cdot \frac{a-2b}{2b} \right) = a \left(\frac{4b^2 - a^2}{4b^2} \right) \cdot \frac{2b}{a^2} : \left(-\frac{a^2 - 4b^2}{2ab} \right) = \\ &= \cancel{a} \cdot \frac{4b^2 - a^2}{\cancel{4b^2}} \cdot \frac{\cancel{2b}}{a^2} : \left(-\frac{a^2 - 4b^2}{2ab} \right) = -\frac{\cancel{4b^2} - a^2}{\cancel{2ab}} \cdot \frac{\cancel{2ab}}{a^2 - \cancel{4b^2}} = 1 \end{aligned}$$

Risolvi l'equazione numerica intera.

Fila A

$$\begin{aligned} & \bullet \quad \frac{5}{4}(6x+2) - 3x = \frac{1}{4}(2x+8) - 2x \\ & \frac{15}{2}x + \frac{5}{2} - 3x = \frac{1}{2}x + 2 - 2x \Rightarrow 15x + 5 - 6x = x + 4 - 4x \Rightarrow 9x + 5 = 4 - 3x \\ & 9x + 3x = 4 - 5 \Rightarrow 12x = -1 \Rightarrow x = -\frac{1}{12} \end{aligned}$$

Fila B

$$\begin{aligned} & \bullet \quad \frac{3}{8}x - \frac{1}{8}(3x-2) = \frac{1}{4} - \frac{2x+1}{2} \\ & \frac{3}{8}x - \frac{3}{8}x + \frac{1}{4} = \frac{1}{4} - \frac{2x+1}{2} \Rightarrow 3x - 3x + 2 = 2 - 8x - 4 \Rightarrow 2 = -8x - 2 \Rightarrow 8x = -4 \\ & x = -\frac{4}{8} \Rightarrow x = -\frac{1}{2} \end{aligned}$$

Fila A

$$\bullet \quad \frac{x-8}{12} + \frac{x+4}{4} = 1 + \frac{x+1}{3}$$

$$x-8+3(x+4)=12+4(x+1) \Rightarrow x-8+3x+12=12+4x+4 \Rightarrow 4x+4=4x+16$$

$$4x-4x=16-4 \Rightarrow 0x=12 \Rightarrow x=\frac{12}{0} \Rightarrow \text{impossibile}$$

$$\bullet \quad \frac{x-8}{12} + \frac{x+4}{4} = \frac{x+1}{3}$$

$$x-8+3(x+4)=4(x+1) \Rightarrow x-8+3x+12=4x+4 \Rightarrow 4x+4=4x+4$$

$$4x-4x=4-4 \Rightarrow 0x=0 \Rightarrow x=\frac{0}{0} \Rightarrow \text{indeterminata}$$

Fila B

$$\bullet \quad 3 + \frac{x-5}{5} = 2 + \frac{1}{5}x$$

$$15+x-5=10+x \Rightarrow x-x=10-15+5 \Rightarrow 0x=0 \Rightarrow x=\frac{0}{0} \Rightarrow \text{indeterminata}$$

$$\bullet \quad 6 + \frac{2(x-5)}{5} = 5 + \frac{2}{5}x$$

$$30+2(x-5)=25+2x \Rightarrow 30+2x-10=25+2x \Rightarrow 20+2x=25+2x$$

$$2x-2x=25-20 \Rightarrow 0x=5 \Rightarrow x=\frac{5}{0} \Rightarrow \text{impossibile}$$